

Elucidation of ‘Cosmic Coincidence’

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Abstract. In the standard cosmological model the dark energy (DE) and nonrelativistic (NR) matter densities are observationally determined to be comparable at the present time, in spite of their greatly different evolution histories. This ‘cosmic coincidence’ enigma – also referred to as the ‘why now?’ problem – relies, by its very definition, on the implicit prior expectation for our ‘typicality’ in the cosmic (expanding) spacetime volume. Otherwise, this conundrum does not exist in the first place. It is shown here that this apparent coincidence could be explained as a *non-anthropic* observational selection effect: for us to be typical observers in the comoving (static) spacetime volume, the cosmic energy budget must contain a non-vanishing DE component. In addition, it is shown that irrespective of the cosmological initial conditions and assuming no ‘new physics’, the Universe is most *likely* to be observed at a time when the conformal Hubble radius, \mathcal{H}^{-1} , attains a maximum. The latter takes place at the epoch when ρ_{DE} and ρ_m , the energy densities of DE and NR matter, respectively, are comparable. Specifically, our presumed ‘typicality’ along the conformal timeline, coupled to a few other plausible assumptions, implies that $R \equiv \rho_{DE}/\rho_m$ is ‘sampled’ from a Beta Prime probability distribution function. *A priori* 68% (95%) confidence range for the ratio is $0.20 < R < 3.46$ ($0.033 < R < 17.20$), with an expectation value of $\bar{R} = 3.5$. These are in agreement with the observationally inferred value, $R_{obs} = 2.23$.

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1 Introduction

The concordance Λ CDM model provides a phenomenologically impressive description of the Universe from super-horizon scales down to supercluster scales with only a few free model parameters. These include the energy density of ordinary baryonic matter, cold dark matter (CDM) and dark energy (DE) [the latter is consistent with a cosmological constant Λ] along with the free parameters of the primordial matter power spectrum and optical depth towards the reionization era. However, the standard Λ CDM model is still marred by several long-standing puzzles.

Whereas the consensus within the community, given currently available observational data, is that the existence of some form of vacuum-like DE seems to be unavoidable, little is known about what it might actually be, e.g. [1–7]. Fundamentally, it is widely believed that the vacuum-like energy density is induced by some unknown slow-rolling scalar field which is not part of the standard model (SM) of particle physics, or effectively by a cosmological constant. Regardless (or not) of its true identity, two major problems are raised by the apparent need for DE.

First, from very general considerations, the zero-mode fluctuations are expected to induce a vacuum energy density at the $O(1)M_p^4$ level, e.g. [1], where M_p is the Planck mass, and the lifetime of a Universe dominated by this vacuum energy is the Planck time. It has long been hoped that due to some (yet-unknown) symmetry principle the vacuum energy density identically vanishes. This all changed in 1998 when it was discovered that cosmic expansion started accelerating at redshift $\lesssim 1$ [8, 9]. Nearly 69% of the cosmic energy budget at present is accounted for by a vacuum-like DE, consistent with a cosmological constant whose energy density is ~ 122 orders of magnitude smaller than the Planck density, in sharp disagreement with theoretical expectations.

A second problem related to the cosmological constant – the Cosmic Coincidence problem (CCP) – is the observational fact that nonrelativistic (NR) matter and vacuum energy densities are of the same order of magnitude at present, whereas their evolution histories are very different [10]. Specifically the DE-to-NR energy ratio scales $\propto (1+z)^{-3}$, and for their ratio to be unity at present it must have been fine-tuned at, e.g. $\gtrsim 80$ decimal places at the GUT epoch. For comparison, the spatial curvature of the Universe must have been fine-tuned at the GUT era at 27 decimal places for the Universe to be nearly flat at present. This latter fine-tuning, known as the ‘flatness problem’, was a major thrust for introducing an early inflationary era to the cosmological model. To illustrate the absurdity of this result, imagine

that NR and DE are not equal at present, but rather that the energy density of DE at present is 20 orders of magnitude smaller than that of NR (regardless of the fact that this tiny value is unlikely to be detected). Even then, the required fine-tuning is at the stupendously large, $\gtrsim 60$ orders of magnitude, level. In other words, the CCP is only one particular aspect of an even more perplexing puzzle – ‘why does DE exist at all?’

An exactly identical reasoning could be applied to the flatness problem, with the inevitable conclusion that spatial curvature should exactly vanish, unless unacceptable level of fine-tuning of the initial conditions is allowed. However, cosmic inflation allow the flattening of space and the subsequent materialization of energy such that space within the observable Universe is very nearly flat.

The CCP, sometimes referred to as the ‘why now?’ problem, can alternatively be posed as follows: Whereas the Hubble radius is generally time-dependent, the length scale, $l_\Lambda = 1/\sqrt{\Lambda}$, associated with the cosmological constant, Λ , is also a constant. The problem is then to explain how is it that specifically at the time of observation, i.e. now, these two scales are comparable.

All these different formulations of the CCP are at best qualitative, citing the number of decimal places of the required precision in setting the initial conditions rather than referring to a quantitative statistical estimate of tension with theory. The reason for that is that theory does not tell us what values should the various cosmological parameters take. Some clues are provided by toy quantum cosmology models or the multiverse as to the likely range of values of the various parameters but these heavily depend on the assumed measure, e.g. [11–15]. Consequently, we find ourselves in the uncomfortable situation that on the one hand the observed cosmic coincidence seems to be puzzling and *a priori* unlikely, but on the other hand this unlikeliness is not well-defined or quantified in a statistically precise fashion.

Proposed resolutions of the CCP range from anthropic considerations, e.g., [1, 16–32] through tracking models [33–37] to interacting DE-DM models [38–43]. Other explanations, e.g. [44–62], abound. While widely believed to be a problem, there is also the opposing view maintaining that there is no problem to begin with, e.g. [63, 64].

In this work an explanation is proposed of the currently comparable values of DE and NR energy densities as a non-anthropic *observational selection effect*: assuming that we are ‘randomly selected’ observers in the comoving (static) – rather than in cosmic (expanding) – spacetime volume it is shown that we are most likely to observe the Universe when $R = O(1)$, where R is the ratio $R \equiv \rho_{DE}/\rho_m$, and ρ_{DE} and ρ_m are the energy densities of DE and NR matter, respectively. This takes place at the minimum total energy density, or equivalently, at the maximum Hubble scale, or minimum (conformal) expansion rate of the Universe, when the Universe spends a relatively large fraction of its conformal lifespan. It should be stressed already from the outset that whereas the assumption that we are randomly sampled in the comoving spacetime volume could be arguably considered to be *ad hoc*, a similar assumption about our typicality in the cosmic frame, (over a sufficiently long, but finite, period of time) lies at the foundations and even existence of the CCP, as well as other problems, e.g. the ‘Boltzmann Brain’ problem of the standard Λ CDM model, e.g. [65, 66]. In the absence of any guiding principles from theory, the ultimate test for any such prior expectation is the naturality or unnaturality of the observationally inferred cosmological parameters given any particular prior assumption. In this work we entertain the plausible possibility that the CCP (among other problems of the Λ CDM model) does not exist if we are randomly ‘drawn’ in the comoving – rather than the cosmic – frame.

The structure of the paper is as follows. In section 2 we advocate the choice of a uniform

prior in the comoving spacetime volume instead of the default analogous prior in the cosmic spacetime volume. This is not essential for our likelihood considerations in section 3 (the latter by no means relies on the arguments that are outlined in section 2) where our analysis is carried out in the ‘dual’ redshift space. In section 4 additional implications are considered beyond the narrow interest of the CCP. We conclude with a summary in section 5.

2 Typicality in the Comoving vs. Cosmic Frame

By their very essence, ‘why now?’ problems in cosmology owe their existence to the implicit assumption that there is *a priori* no preferred time scale in our cosmological models to determine that, e.g. the energy densities associated with DE and NR matter should be comparable specifically at the time of observation. In other words, if anthropic reasoning is excluded, then it is assumed that we are essentially ‘drawn’ from a uniform prior distribution on the time variable. The essence of the ‘why now?’ puzzle is the following; under that assumption, how is it that we are ‘randomly selected’ to observe at just this *special* epoch when DE and NR (energy) densities are comparable in spite of their very different evolution histories. This seems to require stupendously fine-tuned initial conditions, otherwise the comparability of DE and NR (energy) densities at the present time, of all times, seems very *improbable*.

While in general there is a clear sense that this needs to be addressed, the problem itself is not precisely quantified. In particular, the standard cosmological model does not inform us what should the model parameters empirically be, even not statistically. For example, the present-day energy densities of DE and the NR matter can *a priori* take any value and it is only experiment/observation that pins them down. Therefore, it is impossible to quantify, in a conventional statistical way, the severity of the problem in terms of, e.g. statistical ‘tension’ between theory and observations. Rather, one is left with estimates, such as those described in section 1, of the severity of fine-tuning at arbitrary times, e.g. GUT time, or the time when inflation ended, or any other time, in terms of number of decimal places – we simply do not know when exactly the initial conditions have been set, but admittedly these sort of estimates provides some notion of the severity of the problem.

In addition, in asking why ‘now’ of all times are the DE and NR matter densities comparable, one necessarily assumes something about time. The time coordinate is assumed by default to be cosmic time, t . However, a uniform prior on t is not equivalent to a uniform prior on, e.g., conformal time, η , and there is no *a priori* reason to favor the former over the latter, or over other priors on any time coordinate, \tilde{t} . The conformal time, η , will be defined below. In the absence of any additional evidence the question of what prior to use is ultimately decided in practice by comparison of theoretical predictions with observations, guided by the principle that the prior which is most naturally compatible with observations, i.e. with the least amount of apparent fine-tuning, should be favored over others.

One purpose (among others) of the present section is to advocate the choice of a uniform prior on the comoving (static) rather than cosmic (expanding) spacetime volume (which for the time coordinate implies uniform prior on η rather than t). However, as mentioned above, the choice can be made *ad hoc*, much like it is tacitly assumed conventionally (with no clear justification) that t is drawn from a uniform prior (over a sufficiently long, but finite, time period) with no evidence to support this assumption. Quite the contrary, a uniform prior on t is impossible (but rather only over a finite period) and it results in a few well-known puzzling properties of the standard cosmological model, primarily the CCP.

How likely is it for us to make observations of an eternal Universe at the first 14 Gyrs of its existence? This puzzle that pertains to a Universe that has a beginning but no end, such as the Universe we seem to inhabit, implies that there must be a finite probability for our existence within a finite time interval after the Universe ‘starts’. Our prior position along the cosmic timeline, i.e. along the t axis, cannot be uniform in an eternal Universe, thereby making us ‘special’, with pronounced preference to observe the Universe at a measure zero fraction of its eternal lifespan, e.g. closer to its beginning than to its (infinitely remote) end. This ‘naturalness’ issue does not exist in Universes which are both past- and future-eternal on the one extreme, and in Universes that end in, e.g. a Big Crunch, Big Rip [67], or Big Slurp (sudden decay of false vacuum state) [68], on the other extreme.

According to Laplace’s Principle of Insufficient Reason, also known as the Indifference Principle (that was also discussed by Bernoulli, Poincare, and many others), in the presence of maximum ignorance as to how likely is it for an event to take place – all possibilities are equally likely. In Bayesian statistics, this is the simplest non-informative (‘maximum entropy’) prior probability – a uniform distribution. Let us assume that the *true* prior probability of observing the Universe over an infinitesimal time interval, dt , around any given *cosmic* time, t , is $\mathcal{P}(t)dt$, where $\mathcal{P}(t)$ cannot be a non-vanishing constant since according to the concordance cosmological model the Universal expansion is eternal into the future. A new time coordinate, $d\tilde{t} \propto \mathcal{P}(t)dt$, can always be so defined that in terms of which the new probability distribution $\tilde{\mathcal{P}}$ is uniform, i.e. $\tilde{\mathcal{P}}d\tilde{t} = \mathcal{P}(t)dt$. Let us also assume that $\tilde{t} \in [0, T]$, where T is finite. The latter assumption is critical, because it guarantees that observations carried out over a *finite* interval, $\Delta\tilde{t}$, however small it is, have non-vanishing finite probability to actually take place. In this new frame half of the typical observers, i.e. those who were ‘randomly sampled’ from a flat prior in the range $[0, T]$ will observe the Universe over the middle time interval of the entire duration T , i.e. $\tilde{t} \in [T/2, 3T/4]$, 68% percent of them will observe at $\tilde{t} \in [0.16T, 0.84T]$, 95% percent of them will observe over the range $\tilde{t} \in [0.025T, 0.975T]$, etc. We stress once again that the objective of the (admittedly somewhat simplistic) analysis carried out in this section is only to motivate our choice $\tilde{t} = \eta$, and the more detailed and rigorous analysis is relegated to section 3.

In the following, we apply the Principle of Indifference to observations made in a Universe like ours, which is described by the standard cosmological model. The most general homogeneous and isotropic spacetime is described by the Friedmann-Robertson-Walker (FRW) metric

$$\begin{aligned} ds^2 &= -dt^2 + a^2(t) \left[\frac{dr^2}{1 - Kr^2} + r^2(d\theta^2 + \sin^2\theta d\varphi^2) \right] \\ &= a^2(\eta) \left[-d\eta^2 + \frac{dr^2}{1 - Kr^2} + r^2(d\theta^2 + \sin^2\theta d\varphi^2) \right], \end{aligned} \quad (2.1)$$

where t and $\eta \equiv \int \frac{dt}{a}$ are the cosmic and conformal time coordinates, respectively, $a(t)$ is the scale factor, and r , θ and φ are the standard spherical coordinates. The spatial curvature parameter is K . In the ‘t-frame’ [first of Eqs. (2.1)] the lapse function is normalized to unity [69]. It is customary to set $a = 1$ at the present time. In this convention $a = (1 + z)^{-1}$, where z is the cosmological redshift. The second of Eqs. (2.1) describes the same spacetime in a more ‘symmetric’ form where $a(\eta)$ plays the role of an overall conformal factor. In the following we refer to this frame as the ‘ η -frame’. The frame in which $a(\eta)$ is scaled out is known as the ‘comoving frame’, essentially a static spacetime described by the metric $g_{\mu\nu} = \text{diag}(-1, \frac{1}{1-Kr^2}, r^2, r^2 \sin^2\theta)$ where masses rescale as $m_0 \rightarrow m_0 a(\eta)$. According to this

alternative picture the entire cosmic evolution is manifested by growing masses on a static background spacetime. These two alternative pictures are observationally indistinguishable, but as we see below (and elsewhere [70]) a uniform prior on events in the comoving spacetime volume is favorable over a uniform prior on events in the cosmic spacetime volume from the perspective of naturalness and typicality.

The Copernican Principle, referred to as the Cosmological Principle when applied to the cosmological model, embodies the idea that all observers at a given cosmic (or conformal for that matter) time are ‘typical’. However, such a uniform prior on spatial location only results in non-vanishing probabilities if the causally connected volume, i.e. the observable Universe, is finite. This is possible if, e.g. space has a closed geometry or, alternatively, if there is a cosmological horizon that will never be crossed, even in the very remote future. The latter forms when cosmic expansion enters an accelerated phase. As mentioned above, naively applying the Copernican Principle to (cosmic) time itself is problematic in our Universe since the t coordinate is future-infinite. However, if cosmic horizons that result from accelerated expansion do exist then η is finite. We then conclude that the Copernican Principle can be applied to spacetime, not only to space, if cosmic expansion enters an acceleration phase. In that case we can in principle think of uniform priors on each one of the spacetime coordinates, with events parameterized by (η, x, y, z) , where η , x , y and z are independent coordinates in the comoving frame. This four-dimensional uniform distribution – which is the one advocated in the present work – is independent of space expansion because it applies to the comoving coordinates; space expansion is replaced in this alternative description by monotonically growing masses.

The temporal evolution of $a(t)$ is governed by the Friedmann equation

$$H^2 = \frac{8\pi G\rho}{3}, \quad (2.2)$$

where G is the Universal gravitational constant, $H \equiv d\xi/dt$ is the Hubble function where $\xi \equiv \ln a$ has been defined, and $\rho = \sum_i \rho_i$ is a sum over all contributions to the total energy density, including DE. The conformal Hubble function is $\mathcal{H} \equiv d\xi/d\eta = aH$. In the case that these species do not mutually interact and each one individually satisfies the continuity equation, then Eq. (2.2) can be written as

$$H^2(t) = H_0^2 \sum_i \Omega_i e^{-3(1+w_i)\xi}, \quad (2.3)$$

where H_0 is the Hubble constant, Ω_i is the energy density of the i ’th species in critical density units, $\rho_c \equiv 3H_0^2/(8\pi G)$, w_i are the respective equations of state (EOS), and the various Ω_i ’s are subject to the constraint $\sum_i \Omega_i = 1$ (the effective energy density of curvature, $\Omega_k = -K/H_0^2$, included). Clearly, an observation cannot be made by sentient beings (to the best of our current understanding) during the RD era or cosmic inflation, and so our primary focus in this work is on the post-RD era (however, we do not have to rely on this anthropic consideration in the case of the concordance model – this point is referred to below). Spatial curvature is also ignored for the most part of this work, and so $\Omega_{DE} = 1 - \Omega_m$, where Ω_m denotes the energy density of NR matter in critical density units.

Integration of the Friedmann equation, assuming only non-relativistic (NR) matter and and DE (the latter is characterized by an EOS w) results in the conformal time lapse (in Hubble time units) from the big bang to any desired redshift z

$$H_0\eta_z \equiv H_0\Delta\eta(z) = \int_\infty^z \frac{dz'}{\sqrt{\Omega_m(1+z')^3 + (1-\Omega_m)(1+z')^{3(1+w)}}}. \quad (2.4)$$

Applying (2.4) to the case $z = -1$ in a Universe purely made of NR matter results in divergent cosmic lifetime as measured in terms of both t and η . This divergence comes from contributions near $z \rightarrow -1$. However, adding a ‘DE’ component (with $w < -1/3$), that drives an accelerated expansion when the NR contribution becomes subdominant to DE still results in divergent Δt over the lifetime of the Universe, but a finite

$$H_0\eta_{-1}/2 = \frac{\Gamma(1 - \frac{1}{6w})\Gamma[\frac{1}{6}(3 + \frac{1}{w})]R^{\frac{1}{6w}}}{\sqrt{\pi\Omega_m}}, \quad (2.5)$$

where $R \equiv \Omega_{DE}/\Omega_m$, and $\Gamma(x)$ is the Gamma-function. The (dimensionless) conformal lifetime $H_0\eta_{-1}$ is finite due to the existence of an asymptotic future *cosmic horizon*. The limiting case $w = -1/3$ results in a logarithmic divergence. In other words, any species characterized by $w < -1/3$ ‘regularizes’ the (otherwise) infinite (conformal) lifetime of the Universe. The present conformal cosmic age is given by

$$H_0\eta_0/2 = \frac{{}_2F_1(\frac{1}{2}, -\frac{1}{6w}; 1 - \frac{1}{6w}, -R)}{\sqrt{\Omega_m}}, \quad (2.6)$$

where ${}_2F_1$ is the hypergeometric function.

The ratio η_0/η_{-1} is generally a sizable fraction of unity for a wide range of w and Ω_m values as can be seen from Fig. 1. Specifically, applying Eqs. (2.5) – (2.6) to the concordance

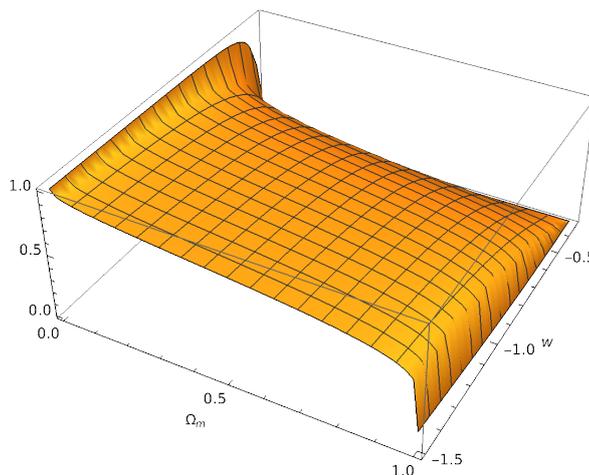


Figure 1. The conformal age of the Universe in conformal lifespan units (η_0/η_{-1}) for a range of Ω_m and w values: Unless $w \rightarrow -\frac{1}{3}$ or $\Omega_m \rightarrow 1$ (where there is basically no horizon and η_{-1} diverges) the ratio η_0/η_{-1} is a sizable fraction of unity, namely we are *a priori* bound to find that we are ‘average’ observers. In the cases of very negative w the Universe is short-lived and we are likely to find ourselves in an old Universe. A similar effect is found when $\Omega_{DE} \rightarrow 1$, i.e. when the Universe exponentially expands and so $\eta_0/\eta_{-1} \rightarrow 1$.

Λ CDM model with $\Omega_m = 0.31$ and $w = -1$ results in $\eta_0/\eta_{-1} = 0.74$, i.e., we are well within the middle 68% confidence interval in terms of η [according to the prescription outlined above Eq. (2.1)], and from this narrow perspective we can indeed be considered average observers; we observe the Universe at a point where 3/4 of the lifetime of our Universe has passed (as opposed to the cosmic frame where we are very atypical observers who just happen to observe

the Universe 14 Gyrs after it formed, essentially at time zero compared to its eternal lifetime). According to the concordance cosmological model η_0 and η_{-1} equal 45 and 61 (conformal) Gyrs, respectively, i.e. we will inevitably hit the ‘end of time’ in 16 (conformal) Gyrs from now. This is shown in Fig. 2 where $H_0\eta_z$ is plotted for a range of redshifts. The shaded light blue region corresponds to the middle 68% range of $H_0\eta_z$ ($-0.42 < z < 21.9$), and the median takes place at $z = 1.54$. The present day $H_0\eta_0$ value falls within this range. For reference, the middle 95% range lies within the range ($-0.91 < z < 528$), essentially excluding the radiation-dominated (RD) era (radiation-matter equality takes place at $z_{eq} \approx 3400$ and the *a priori* betting odds for us to be randomly drawn from a uniform distribution in η at the RD era are less than 7 in a thousand). In other words, given that $\Omega_m = 0.31$, and assuming that we are typical observers in the comoving frame excludes the possibility that we are formed at the RD era at 99% confidence with no recourse to anthropic reasoning, biological evolution modeling, etc.

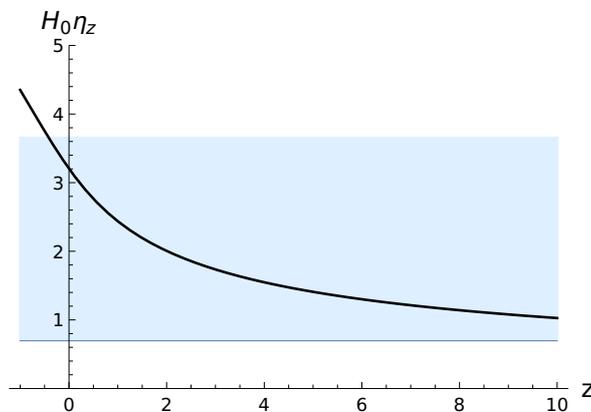


Figure 2. The conformal time lapse in Hubble time units, $H_0\eta_z$, is plotted for a range of redshifts, assuming the concordance Λ CDM model parameters. The middle 68% range of $[H_0\eta_\infty, H_0\eta_{-1}]$ is highlighted as a shaded light blue region. The present day $H_0\eta_0$ value falls within this ‘confidence’ region.

The conclusion of all this is that whereas the assumption that we are randomly positioned in the cosmic (expanding) spacetime volume of a Universe that has a beginning but no end results in glaring typicality problems, *the alternative hypothesis that we are observers who are ‘randomly selected’ from a uniform prior distribution in the comoving spacetime volume results in no such naturalness problems*, provided that at some point the Universal expansion (as described in the expanding frame) accelerates. In other words, for us to be typical in the comoving spacetime volume plausibly requires DE to exist. In the next section we apply a uniform prior in η to the CCP formulated in redshift-space.

We conclude this section with a demonstration of the utility of the assumption of ‘typicality’ in the comoving spacetime volume. Specifically, and with an eye towards the discussion in section 3, useful bounds on the ratio R are derived (with no recourse to any prior knowledge) under the single assumption that we are randomly situated in the comoving spacetime volume of a Universe that has a finite conformal lifetime (i.e. it ends its evolution with an accelerated expansion phase). Specifically, we customize an approach, that was popularized by Gott originally in the context of the ‘Doomsday Argument’ [71], to the case at hand. This, essentially typicality (sometimes referred to as ‘mediocrity’) argument, e.g. [12, 71], is

not uncontroversial, e.g. [26, 72–74], but it is nevertheless useful, in the spirit of the present section, to derive these constraints from the assumption that we are typical in the comoving frame. This is especially warranted in the context of the cosmic ‘why now?’ problem, which is at the focus of the present work; as mentioned above, in its very essence this problem arises due to the tacitly made assumption that we are typical observers along the cosmic timeline t , or in other words it is implicitly assumed that we are typical in the expanding – cosmic – frame, i.e. why are Ω_m and Ω_{DE} comparable at present (t_0) while they are much different at any other time; what makes the present time particularly special (given the prior expectation that we are typical)?

Assuming that we are random observers along the Universal η -axis then there is 50% chance that we are in the middle 50% interval of the entire Universal lifespan. The two extremes of this confidence interval are at 1/4 and 3/4 the way from the big bang, $\eta_\infty = 0$, to the end of time, η_{-1} . If we happen to observe the Universe at the one extreme, then $\eta_0 = \frac{\eta_{-1}}{4}$. On the other hand, if we observe the universe at the other extreme of this interval, then $\eta_0 = \frac{3\eta_{-1}}{4}$. Inverting these relations we obtain $\frac{4}{3}\eta_0 < \eta_{-1} < 4\eta_0$ at the 50% confidence level. Applying a similar reasoning to obtain the 68% confidence interval results in $1.19\eta_0 < \eta_{-1} < 6.25\eta_0$. Employing Eqs. (2.5) – (2.6) we obtain that $R \in [0.00186, 2.55]$ and $R \in [1.128 \times 10^{-4}, 10.70]$ at the 50% and 68% C.L, respectively. These estimates, which rely on no prior measurements or data, and only assume that we are typical observers along a *finite* η axis as well as the validity of Eqs. (2.5) – (2.6) (i.e. the assumption that the Friedmann equation adequately describes the cosmic evolution and that typical observers are found in a Universe largely dominated by either NR matter, DE, or both) should be compared to the observationally inferred value. According to the concordance Λ CDM model $\Omega_m = 0.31$ and so $R_{obs} = 2.23$. Since these probability estimates rely on our *a priori* ignorance as to our actual position in *spacetime*, rather than on any prior knowledge, the existence of DE could have been predicted long before the discovery of the accelerated expansion in 1998 [8, 9], if only we were assumed to be ‘randomly sampled’ from the comoving static spacetime volume.

It is startling that applying the ‘Principle of Indifference’, basically assigning uniform prior probability to our position in the comoving spacetime volume, is sufficient to conclude that a DE component (described by $w < -1/3$) must exist. The observed ratio, R_{obs} is then found to be consistent with the 68% confidence interval that is retrodicted based on this remarkably modest assumption. However, it has been claimed that the seemingly innocuous assumption of typicality grants us an (perhaps unwarranted) ‘enormous leverage’ [66, 72]. On the other hand, and specifically following the argument above Eq. (2.1), the assumption of typicality is always correct is some specific time coordinate \tilde{t} . In the present work we advocate that this time coordinate is the conformal time, η .

All these considerations and results lend support (but are not essential) to our proposal to replace the implicit assumption of typicality in the cosmic frame with a more ‘realistic’ typicality in the comoving frame. Again, it is certainly legitimate to *ad hoc* posit a prior on η in the same fashion that it is conventionally and tacitly done for t (over a sufficiently long period after the big bang).

3 Observational Likelihood Function

Proposed anthropic solutions to the CCP invariably involve considerations of the growth of structure. However, for our non-anthropocentric arguments it will be sufficient to work at the smooth and homogeneous background level. The analysis of section 2 was restricted to the

time-domain. In the present section we carry out our probabilistic estimates in the ‘dual’, redshift-domain, taking advantage over the entire available redshift range that provides a better leverage and constraints on the energy density parameters. The two are related by gravitation, specifically via the Friedmann equation in our case.

Before turning to the construction of the likelihood function in redshift space it is perhaps constructive to examine a well-known mechanical system where a similar observational selection effect is most easily grasped – that of a one-dimensional undamped harmonic oscillator. If the periodic motion of an oscillator is randomly sampled in time, it is more likely to be found near the extreme ends of the range of its motion (where it momentarily comes to an halt just before turnaround) than near the equilibrium point where the motion is the fastest. A straightforward calculation shows that it only spends 1/3 of each oscillation period (i.e. of the number of random samplings) in the middle half of its range of motion between the extreme points. The rest 2/3 is spent in the far half of its trajectory, closer to the end points. In this classic observational selection effect example the oscillator instantaneous velocity plays an analogous role to the one played by \mathcal{H} , the conformal Hubble expansion rate, in the cosmological context as we see below.

Back to cosmology, following the discussion in section 2, and adopting a uniform prior in the comoving spacetime volume rather than expanding volume, our ‘position’ along the timeline is ‘drawn’ from the prior

$$\mathcal{P}(\eta)d\eta = C d\eta, \quad (3.1)$$

where $C^{-1} \equiv \eta_{-1}$ is the lifetime of the Universe as defined by Eq. (2.5). In terms of cosmic time, t , this prior corresponds to a probability distribution $\mathcal{P}(t) \propto a^{-1}$. In an expanding Universe that grows with no limit this distribution favors ‘small’ scale factors, a , i.e. earlier cosmic times in an eternal Universe. In other words, although our Universe is likely to spend essentially an infinite time in an exponentially, DE-dominated, phase, we still happen to observe it at just the transition phase between matter and DE-domination. This is because the odds are systematically tilted towards smaller scale factors, i.e. earlier times, due to the probability distribution $\mathcal{P}(t) \propto a^{-1}$, as is proposed by Eq. (3.1), that effectively imposes an exponential cutoff once the accelerated expansion phase begins; we observe the Universe at the latest (practically) possible time in an infinite Λ CDM Universe. This provides a heuristic argument, under the assumption of flat prior in η , for the fact that our observation is made during the transition epoch between NR matter and DE; while it is true that the Universe spends an infinite time period in the DE-dominated era the probability of making an observation during this period is exponentially suppressed. Since the transition takes place a finite (cosmic) time after then big bang then it stands to reason that observations are most likely to take place a finite (cosmic) time after the big bang in spite of the fact that the Universe is eternal.

Consider the concordance Λ CDM model in which the cosmic energy budget is dominated by a vacuum-like energy density in the far future. The conclusion of the following discussion is robust to reasonably changing the EOS describing the asymptotic future to any constant $w < -1/3$ or even slowly evolving EOS. We will see below that a temporally random observer (in η) is most likely to make observations when $R = O(1)$. We stress once again that matter perturbations over the smooth background are not required to satisfy any particular conditions (or even to exist) for the following considerations to apply.

Observing the Universe over a finite (conformal) time interval $\Delta\eta$ has a non-vanishing probability *only if* η is finite, i.e. the probability density function is normalizable. As men-

tioned above, our very existence a finite (cosmic) time after the big bang in an eternal Universe is already very puzzling if we were to be drawn from a *uniform* prior on t unless anthropic considerations are employed. However, in terms of conformal time, η , the scale factor in the Λ -dominated phase is readily obtained by integration of Eq. (2.2), $a \propto (\eta_* - \eta)^{-1}$, where $\eta_* > 0$ is a constant of integration. In that case $0 < \eta < \eta_*$ is always finite, as is already evident from the discussion in section 2.

From Eq. (2.2) it follows that $d\eta = (\frac{8\pi G\rho}{3})^{-\frac{1}{2}} e^{-\xi} d\xi$, and so if we consider a flat prior in η for observing the Universe, then the probability functions $\mathcal{P}(\eta)$ and $P(\xi|\{\Omega_i\})$ are related via

$$\mathcal{P}(\eta)d\eta = Cd\eta = P(\xi|\{\Omega_i\})d\xi \propto \frac{d\xi}{\mathcal{H}(\xi; \{\Omega_i\})} \propto \frac{e^{-\xi}d\xi}{\sqrt{\rho}}, \quad (3.2)$$

where \mathcal{H} is the conformal Hubble function, $\{\Omega_i\}$ stands collectively for the energy densities of the various contributions to the total energy budget (in critical density units), and $P(\xi|\{\Omega_i\})\Delta\xi$ is the conditional probability of making an observation over an interval $\Delta\xi$ around ξ ($a \equiv e^\xi$) given the parameter values $\{\Omega_i\}$. Therefore, a flat prior on observing the Universe at around any given η , i.e. $\mathcal{P}(\eta) = C$, *generally* does not correspond to a flat prior on ξ , since $P(\xi|\{\Omega_i\}) \propto \mathcal{H}^{-1}$ varies with ξ and is the largest (and correspondingly more likely for the Universe to be observed at) when the conformal expansion rate attains a minimum, i.e. $d\mathcal{H}/d\xi = 0$, which corresponds by virtue of the Friedmann equation to $\mathcal{W} = -1/3$, where \mathcal{W} is the density-weighted average EOS. In our case $\mathcal{W} = w\Omega_{DE}$, i.e. $\Omega_{DE} = -1/(3w)$. Since $\mathcal{W} = -1/3$ is a mixture of both NR matter and DE then it follows that $w < -1/3$, in accordance with the discussion following Eq. (2.5). We reiterate that the EOS of the individual species are assumed to be fixed constants. Since \mathcal{W} drops with the expansion of the Universe it follows from the currently observed value, $\mathcal{W} = -2/3$, that we observe the Universe past its lowest conformal expansion rate, as is also discussed below Eq. (2.6).

Applied to w CDM, Eq. (3.2) reads

$$P(\xi|\Omega_m)d\xi \propto \frac{d\xi}{\sqrt{\Omega_m e^{-\xi} + (1 - \Omega_m)e^{-(1+3w)\xi}}}, \quad (3.3)$$

where $w < -1/3$. It follows from Eq. (3.3) that $P(\xi|\Omega_m)$ peaks at $\xi = 0$ ($a = 1$) if $\Omega_m = (1 + 3w)/(3w)$ [which implies that if $w \lesssim -1/3$, as we have shown in section 2 to be necessary, then $\Omega_m \lesssim 1$, i.e. DE is never negligible at the most likely observable value]. The special case $\Omega_m = 0$ and $w = -1/3$ corresponds to a flat distribution. This is the single case where both $\mathcal{P}(\eta)$ and $p(\xi)$ are flat, and the corresponding spacetime is that of Milne's empty spacetime [75–77]. The lesson is that once $\rho \neq 0$ and the energy budget contains a DE component with $w < -1/3$ then $P(\xi|\Omega_m)$ has a peak and is normalizable (first indication for the latter condition have been discussed in section 2 and will be further discussed below). In other words, $P(\xi|\Omega_m)$ peaks solely due to the effect of gravitation, which is accounted for in the present context by invoking of the Friedmann equation in Eq. (3.2).

Specifying to the case $w = -1$, i.e. assuming that DE is a pure cosmological constant, and viewing Eq. (3.3) as a *normalized* distribution in ξ , we obtain

$$\tilde{P}(\xi|\Omega_m) = \frac{3\sqrt{\pi}\Omega_m^{\frac{1}{3}}(1 - \Omega_m)^{\frac{1}{6}}}{\Gamma(\frac{1}{3})\Gamma(\frac{1}{6})} \frac{1}{\sqrt{\Omega_m e^{-\xi} + (1 - \Omega_m)e^{2\xi}}}, \quad (3.4)$$

where $\int_0^\infty \tilde{P}(\xi|\Omega_m)d\xi = 1$. The steep decline of the probability function towards zero in the limits $\Omega_m \rightarrow 0$ and $\Omega_m \rightarrow 1$ that is evident from Eq. (3.4) is visually illustrated in Fig. 3.

The fact that when either $\Omega_m \rightarrow 0$ or 1 the probability function vanishes basically implies that the fundamental question of why does DE exist at all (section 1) is basically answered; under the assumption that we are typical along the η -axis, its maximum lifetime η_{-1} has to be finite (section 2), and observing the Universe is then only sensible if $\int_0^\infty \tilde{P}(\xi|\Omega_m)d\xi = 1$, which implies that DE is ought to exist. In fact, convergence of the integral only requires the existence of a phase of accelerated expansion, i.e. $w < -1/3$. Again, this is not independent of a similar conclusion arrived at in section 2.

Interpreted in a Bayesian fashion, and applying Bayes Theorem to the case discussed here, results in a few useful results. The Bayes Theorem reads

$$P(\xi|\Omega_m) = \frac{P(\Omega_m|\xi)P(\xi)}{P(\Omega_m)}. \quad (3.5)$$

Considering $\tilde{P}(\xi|\Omega_m) = P(\xi|\Omega_m)P(\Omega_m)$, the various probability functions can be readily read off. These are

$$P(\Omega_m|\xi) \propto \frac{\Omega_m^{\frac{1}{3}}(1 - \Omega_m)^{\frac{1}{6}}}{\sqrt{\Omega_m e^{-\xi} + (1 - \Omega_m)e^{2\xi}}}, \quad (3.6)$$

$$P(\Omega_m) = \frac{\Omega_m^{\frac{1}{3}}(1 - \Omega_m)^{\frac{1}{6}}}{B(\frac{4}{3}, \frac{7}{6})}, \quad (3.7)$$

$$P(\xi) = \frac{1}{\xi_2 - \xi_1}, \quad (3.8)$$

and $P(\xi|\Omega_m)$ follows from Eqs. (3.4) and (3.7), and $B(\alpha, \beta) \equiv \Gamma(\alpha)\Gamma(\beta)/\Gamma(\alpha + \beta)$ is the Beta function of two parameters α and β . The parameters ξ_2 and ξ_1 are upper and lower cutoffs on ξ which are computationally irrelevant insofar they are sufficiently distant from $-\xi_0 \equiv \frac{1}{3} \ln[2R]$ (i.e. $\xi_1 \ll \xi_0 \ll \xi_2$) where the distribution $P(\xi|\Omega_m)$ peaks. Interestingly, it follows from Eq. (3.7) that Ω_m is ‘sampled’ from a ‘Beta distribution’ and by virtue of the constraint $\Omega_{DE} = 1 - \Omega_m$ (ignoring curvature and radiation) it then follows that Ω_{DE} is drawn from an identical distribution. The flat prior on ξ [Eq. (3.8)] follows from applying the ‘principle of transformation groups’, merely a generalization of the principle of indifference, to the scale factor a . The logarithm of a , i.e. ξ , becomes a ‘location parameter’, for which the principle of transformation groups suggests a unique, uniform, prior.

Fig. 3 delineates the probability distribution $P(\Omega_m|\xi)$ in the Λ CDM model for the cases $a = 0.5, 1$ and 2 , respectively. It is apparent from the plot that the distribution curves skew towards higher (lower) Ω_m values as a is larger (smaller) than unity. This is easy to understand as Ω_m are the present day values, i.e. it is normalized to $a = 1$. For example, if it is given that an observation is made at $a = 2$ the observed value $\Omega_m(a = 2)$ must match to the most probable value but since $\Omega_m(a) = \Omega_m a^{-3}/(\Omega_m a^{-3} + \Omega_{DE})$, i.e. $\Omega_m(a)$ decays in an expanding Universe, then the inferred $\Omega_m(a = 1) \equiv \Omega_m$ is larger. Analogously, the observed Ω_m at present corresponds to larger values at any $a < 1$.

Since Ω_m and Ω_{DE} appear with small powers in $P(\Omega_m)$ the distribution function is broad, weakly dependent on, e.g. Ω_m . To obtain the 68, 95 and 99% confidence intervals we calculate the area under the curve $P(\Omega_m)$ leaving out the 16, 2.5 and 0.5% distribution tails on each side of the distribution. In these cases we obtain

$$\begin{aligned} \Omega_m &\in [0.22, 0.84] \quad \& \quad \Omega_{DE} \in [0.16, 0.78]; \quad 68\%C.L. \\ \Omega_m &\in [0.055, 0.968] \quad \& \quad \Omega_{DE} \in [0.032, 0.945]; \quad 95\%C.L. \\ \Omega_m &\in [0.016, 0.992] \quad \& \quad \Omega_{DE} \in [0.008, 0.984]; \quad 99\%C.L., \end{aligned} \quad (3.9)$$

assuming no prior knowledge about cosmological parameters, other than that $\{\Omega_i\}$ are ‘sampled’ from $(0,1)$, and that we observe at the ‘present’ time, i.e. at $\xi = 0$. For reference, the concordance values, $\Omega_m = 0.31$ and $\Omega_{DE} = 0.69$, lie within the 68% confidence interval.

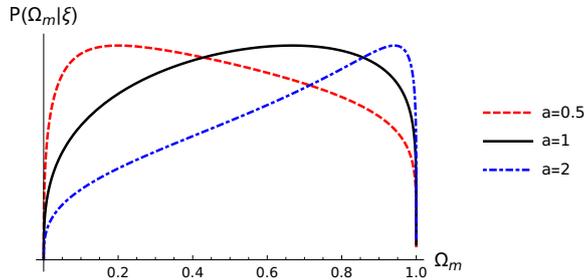


Figure 3. Shown is $P(\Omega_m|\xi)$, i.e. given that we observe at $a = 0.5, 1$ and 2 , what are the probability functions of Ω_m (dashed red, continuous black and dot-dashed blue curves, respectively).

To illustrate the robustness of our conclusion we show $P(\xi|\Omega_m)$ at $\Omega_m = 0.31$ in Fig. 4. Once again it is seen that the Universe is most likely to be observed when $R = O(1)$ within a relatively narrow range in ξ [$P(\xi|\Omega_m)$ has a clear peak and width while $\mathcal{P}(\eta)$ is constant and featureless]. Even though Fig. 4 compellingly illustrates that the concordance Λ CDM is consistent with observations made at (or near) the peak of the probability function, it still remains to be checked for the *look-elsewhere effect*; how unique is the concordance Λ CDM in having the probability function peak at the present time? To test for the robustness of this result we consider the likelihood of observing the Universe at other values of ξ . Employing $\Omega_m(a) = \Omega_m a^{-3}/(\Omega_m a^{-3} + \Omega_{DE})$ and $\Omega_{DE}(a) = 1 - \Omega_m(a)$ where $\Omega_m \equiv \Omega_m(a = 1)$, and $\Omega_{DE} \equiv \Omega_{DE}(a = 1)$, and specifically considering the cases $a = 1/(20)$ and $a = 20$ results in Fig. 5.

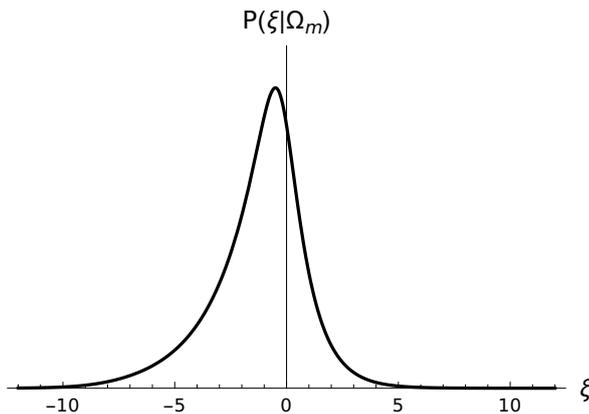


Figure 4. Likelihood for observation in the Λ CDM model with $\Omega_{DE} = 0.69$ and $\Omega_m = 0.31$. Within a few decades in a we expect that at the time of observation $R = O(1)$.

To make contact with section 2, and in particular to compare the present analysis to the constraints obtained in section 2 on R , we recast Eq. (3.7), using $\Omega_m = (1 + R)^{-1}$, i.e.

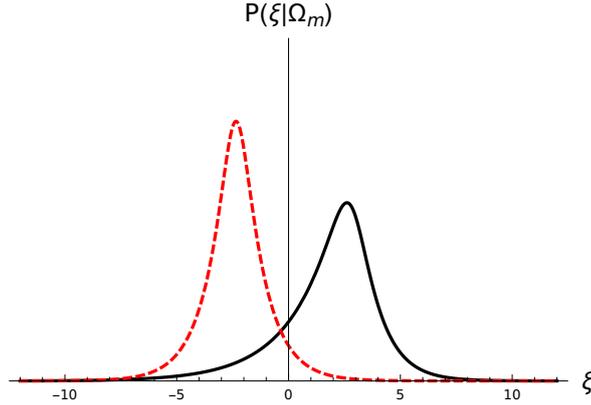


Figure 5. As in Fig. 4 but with the Λ CDM parameters rescaled to the cases $a = 1/20$ (black solid line) and $a = 20$ (red dashed line). Comparison with Fig. 4 illustrates that it is more likely to make a first observation of the Universe at the present time than it was at, e.g. $z = 19$, or will be when the Universe grows twenty times larger than its current size.

$P(\Omega_m)d\Omega_m \propto P(R)dR$ where

$$P(R) = \frac{27\sqrt{\pi}}{2\Gamma(\frac{1}{3})\Gamma(\frac{1}{6})} R^{\frac{1}{6}}(1+R)^{-\frac{5}{2}}. \quad (3.10)$$

This is a Beta Prime distribution (sometimes referred to as Beta distribution of the Second Kind or Inverted Beta distribution), with a probability distribution function of the form $P(R) = R^{\alpha-1}(1+R)^{-\alpha-\beta}/B(\alpha,\beta)$, with $\alpha = 7/6$ and $\beta = 4/3$.

It is clear from its functional dependence that tight (*a priori*) upper limits on R are placed once $R \gg 1$. This probability distribution function for the dimensionless $R = \Omega_{DE}/\Omega_m$ results in the 50%, 68% and 95% confidence intervals $0.32 < R < 2.17$, $0.20 < R < 3.46$ and $0.033 < R < 17.20$, respectively. The latter have been extracted from $P(R)$ following the prescription outlined above Eq. (3.9). The average value $\bar{R} = 7/2$ is obtained from $\bar{R} = \int_0^\infty RP(R)dR$. The probability distribution function $P(R)$ is shown in Fig. 6

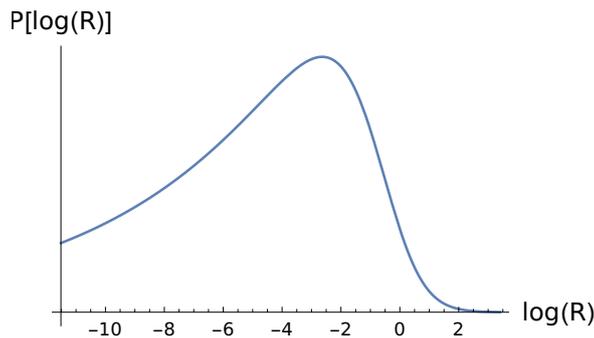


Figure 6. Shown is $P(R)$, the probability distribution function for $R = \Omega_{DE}/\Omega_m$, at the present time, on a semilogarithmic plot.

Again, in arriving at these confidence ranges no prior knowledge other than a uniform prior on η , in addition to the validity of the Friedmann equation (neglecting radiation and

curvature and positing that $w = -1$), has been assumed. The observationally inferred value, $R_{obs} = 2.23$, falls within the 68% confidence range.

As mentioned below Eq. (3.3), it is demonstrated in Figs. 4-5 that $P(\xi|\Omega_m)$ is narrower than a flat $\mathcal{P}(\eta)$, and in addition the former peaks at around $\xi = 0$ only in case that Ω_m is comparable to Ω_{DE} ; the CCP consequently arises from the (implied) expectation that we should *a priori* be positioned randomly along the *cosmic* timeline (impossible in a Universe with an infinite lifetime) while a more sensible choice would favor a uniform prior on η . In other words, our default expectation that we are ‘randomly sampled’ from a uniform prior distribution on the expanding spacetime volume, rather than the comoving spacetime volume, inevitably results in the CCP, along with a few other hitherto puzzling (and even disturbing [65, 66]) properties of the standard cosmological model.

4 Other Implications

We point out two additional implications. First, so far we have considered the background evolution; we now discuss an implication for the growth of perturbations in the linear regime on sub-horizon scales (ignoring relativistic corrections). In the standard cosmological model, the matter overdensity, $\delta_\rho \equiv \delta\rho/\rho$, scales linearly with a , i.e. grows linearly with the scale factor inasmuch as the cosmic evolution is dominated by NR matter, e.g. [78]. This steady growth stops once DE takes over. In our Universe, the latter transition took place fairly recently. This well-understood growth dynamics, by itself, provided the basis for the proposed anthropic resolution of the CCP, e.g. [16, 17]. However, as mentioned above already, rather than approaching the CCP via the growth rate of linear perturbations we attempt at employing the non-anthropic observational selection effect, that was described above, in explaining the current growth rate of linear density perturbations.

The overdensity, δ_ρ , and scalar metric perturbations Φ in the gravitational potential, are related via the Bardeen equation and the relativistic generalization of the gravitational Poisson equation, e.g. [79, 80]

$$\begin{aligned} \Phi'' + 3(1 + \mathcal{W})\mathcal{H}\Phi' + k^2\Phi &= 0 \\ \mathcal{H}\Phi' + \left(\mathcal{H}^2 + \frac{k^2}{3}\right)\Phi &= -\frac{1}{2}\mathcal{H}^2\delta_\rho, \end{aligned} \quad (4.1)$$

where primes denote derivatives with respect to conformal time, k is the Fourier mode of the perturbation, and \mathcal{W} is the effective EOS of the *total* cosmic energy. Combining the Bardeen equation with the temporal derivative of the Poisson equation to eliminate the $\propto \Phi''$ term we obtain

$$\mathcal{H}'(\Phi' + 2\mathcal{H}\Phi + \mathcal{H}\delta_\rho) + k^2\left[\frac{\Phi'}{3} - \mathcal{H}\Phi\right] + \frac{1}{2}\mathcal{H}^2\delta'_\rho = [2 + 3\mathcal{W}]\mathcal{H}^2\Phi'. \quad (4.2)$$

In the long wavelength limit, $k/\mathcal{H} \ll 1$, and at the most likely time for the Universe to be observed at, i.e. when $\mathcal{H}' = 0$ (as concluded in section 3), the first two terms on the left hand side drop and we are left with $\delta'_\rho = 2(2 + 3\mathcal{W})\Phi'$. This implies that insofar $\mathcal{W} = -2/3$ the overdensity δ_ρ attains a maximum at the most likely time for observation. For example, in case of the concordance cosmological model ($\Omega_m \lesssim 1/3$ and $\Omega_{DE} \gtrsim 2/3$) $\mathcal{W} \approx -2/3$ we observe the Universe at its maximum overdensity on super-Hubble scales. Consequently, the most likely state of the Universe to be observed at is also the most clumpy one (at linear order). This sheds light on the anthropic approach to the CCP from a different perspective;

the observed Universe is the one we see not necessarily because we require sufficient clustering for our existence, but rather because, statistically, the Universe is observed in its most likely state. This also turns out (but not *a priori* required) to be its clumpiest state (to first order and in the long wavelength limit), and is then likely to harbor sentient life.

The second implication is that from the perspective adopted in the present work, a spatially closed Universe (with $\Omega_k < 0$) which contains only NR matter will most probably be observed at turnaround, when its expansion rate, $\mathcal{H}(\xi) = H_0 \sqrt{\Omega_m e^{-\xi} - |\Omega_k|}$, ‘momentarily’ vanishes. A similar conclusion holds for a Universe that contains only matter and a *negative* cosmological constant, when its expansion rate, $\mathcal{H}(\xi) = H_0 \sqrt{\Omega_m e^{-\xi} - |\Omega_\Lambda| e^{2\xi}}$, vanishes.

5 Discussion

The Copernican Principle applied to the standard cosmological model (i.e. the Cosmological Principle) proved to be a powerful simplifying assumption which to date has been found to be remarkably consistent with observations; on cosmological scales, there simply is no special point in space. However, when applied to the *cosmic* timeline, the Principle of Indifference – the expectation that we are typical observers along the cosmic history – results in a series of paradoxes and naturality problems.

The Cosmic Coincidence puzzle of the near equality of DE and NR matter contributions to the cosmic energy budget at present, arises from the prior expectation that, barring anthropic considerations, there seems to be no apparent reason (other than fine-tuned initial conditions) for their equality specifically at the time of observation, i.e. the present time, given their very different evolutionary histories.

Irrespective of ones judgement of the scientific basis for the frequently-adopted assumption of typicality in cosmology, multiverse, planetary science, etc., the ‘why now?’ problem associated with DE, by its very essence, relies on the implicit assumption of typicality; ‘why now’ if no other time is *a priori* less favorable to witness the currently observed near equality of DE and NR? In other words, should the principle of typicality be discarded then the ‘why now?’ problem automatically goes a way with it. In addition, as discussed in section 2 temporal typicality is merely a matter of definition (under certain plausible assumptions), at least in the cosmological context; What time coordinate are we typical at?

However, as argued in the present work, accepting the cosmic coincidence as an *a priori* puzzling property of the standard cosmological model that needs to be explained, and assuming a flat prior for observing the Universe at any conformal time interval $\Delta\eta$ around η seems to naturally explain the (otherwise) surprising comparability of DE and NR matter at present. Specifically, a flat prior on η does not correspond to a flat prior on ξ . It is an immediate consequence of the Friedmann equation that the probability distribution, $P(\xi|\Omega_m)$, peaks at the minimum of the total energy density (as defined in the comoving frame). This corresponds to $R = O(1)$ and to the maximum (conformal) Hubble radius, \mathcal{H}^{-1} .

The essence of these considerations is that, in general, the total energy density $a^2\rho(a)$ (in the comoving frame) attains a minimum when the energy densities of DE and NR matter are comparable. Without having to introduce new fields or to change the dynamics of concordance Λ CDM, this provides a non-anthropocentric resolution to the CCP; it implies that at a given randomly chosen finite interval $\Delta\xi$ the Universe is most likely to be found in a state where $R = O(1)$. All we need for that to work is to adopt the prior assumption that we are typical observers in the comoving spacetime volume.

As discussed in section 4, the conclusion that the Universe is most likely to be observed at its lowest $a^2\rho(a)$, i.e. at its lowest (conformal) expansion rate, \mathcal{H} , elucidates the role played by anthropic considerations in proposed solutions of the problem. This is synonymous to overdensities at the long wavelength limit attaining their maximum value (to first order) when the background (conformal) density is minimized and the effective EOS is $\mathcal{W} = -2/3$, i.e. at an epoch very similar to the one we live in at the present era. The upshot is that a uniform prior for observing the Universe at any given time is in general consistent with the anthropic principle applied to this particular CCP. In other words, rather than imposing the requirement that the state of the Universe should be conducive to the existence of sentient beings to observe it, one can make the more modest (and perhaps natural) requirement that we are *a priori* equally likely to observe the Universe at any spacetime event in the four-dimensional *comoving* frame.

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