On gravitational preheating

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Abstract

We consider dark matter production during the inflaton oscillation epoch. It is conceivable that renormalizable interactions between dark matter and inflaton may be negligible. In this case, the leading role is played by higher dimensional operators generated by gravity and thus suppressed by the Planck scale. We focus on dim-6 operators and study the corresponding particle production in perturbative and non–perturbative regimes. We find that the dark matter production rate is dominated by non–derivative operators involving higher powers of the inflaton field. Even if they appear with small Wilson coefficients, such operators can readily account for the correct dark matter abundance.

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1 Introduction

The nature of dark matter (DM) remains an outstanding mystery of modern physics. The null DM direct detection results motivate one to explore the possibility that dark matter has feeble interactions, in which case it does not reach thermal equilibrium with the environment. Therefore, its abundance is sensitive to the production mechanism. One of such mechanisms is provided by gravity, which can efficiently produce particles in non–adiabatic environments.

In the absence of any non–gravitational couplings, the expansion of the Universe is itself a source of particle production [1],[2]. For example, the equation of motion (EOM) for a momentum mode χ_k of a free scalar in the Friedmann Universe with the metric $ds^2 = a^2(\eta)(d\eta^2 - d\mathbf{x}^2)$ reads [3]

$$\chi_k'' + \omega_k^2 \chi_k = 0, \tag{1}$$

where

$$\omega_k^2 = k^2 - \frac{a''}{a}(1-6\xi) + m_\chi^2 a^2 , \tag{2}$$

a is the scale factor, ξ is the non–minimal coupling to gravity [4] and the prime denotes differentiation with respect to conformal time η. Time variation of ω_k is non–adiabatic if $\omega_k'/\omega_k^2 \gtrsim 1$, which implies particle creation due to expansion. For low k and conformal coupling $\xi = 1/6$, this is equivalent to $a^0/a^2 = H$ & m_χ such that particles lighter than the Hubble rate H are constantly created. The effect can be even stronger for non–conformal ξ. The accumulated abundance of χ can constitute dark matter [3, 5], depending on m_χ and its self–interaction.

Such particle production can be viewed in terms of the scalar field condensate hχ^2i. Light scalars are subject to quantum fluctuations of order H [6] so that a semi–classical field χ experiences a random walk. As a result, a significant hχ^2i can accumulate, for example, by the end of inflation and play the role of dark matter [7],[8]. Again, this mechanism is purely gravitational. The consequent dark matter distribution is not correlated with the inflaton fluctuations, therefore this possibility is subject to strict isocurvature constraints [8].

In this work, we focus on other aspects of particle production due to gravitational effects. Specifically, gravity is believed to generate couplings between different sectors of the theory as long as these are consistent with gauge symmetries. The corresponding operators may be non–renormalizable and thus suppressed by the Planck scale. Nevertheless, they can play an important role in dark matter production. This was recently emphasised in [9],[10], where the effects due to tree level graviton exchange were considered.

We study dark matter production during the inflaton oscillation phase, which sets in immediately after inflation and creates a non–adiabatic environment [11, 12, 13]. Using the effective field theory approach, we focus on the leading gravity–induced dim–6 operators assuming that the renormalizable couplings between the inflaton and dark matter vanish. If DM is feebly interacting, its eventual abundance is determined by the number of DM quanta produced at this "preheating" stage. To this end, we identify the dominant operator and study whether it can be responsible for the correct dark matter abundance.

2 The set–up

Consider the possibility that the renormalizable couplings between the inflaton φ and dark matter s are zero or negligibly small. Then, the $\varphi - s$ interaction can be described by a series of higher dimensional operators generated by gravity and thus suppressed by the Planck scale $M_{\rm Pl}$. Let us assume for simplicity that these operators exhibit an approximate $\varphi \to -\varphi$ symmetry such that the lowest operator dimension is six:[1]

$$\Delta\mathcal{L}_6 = \frac{C_1}{M_{\rm Pl}^2}(\partial_\mu \phi)^2 s^2 + \frac{C_2}{M_{\rm Pl}^2}(\phi\partial_\mu\phi)(s\partial^\mu s) + \frac{C_3}{M_{\rm Pl}^2}(\partial_\mu s)^2\phi^2 - \frac{C_4}{M_{\rm Pl}^2}\phi^4 s^2 - \frac{C_5}{M_{\rm Pl}^2}\phi^2 s^4 , \tag{3}$$

where we have replaced the covariant derivatives with the partial ones. The inflaton field with mass m_φ is assumed to have either (locally) quadratic or quartic potential, while the dark matter mass m_s is taken to be negligible compared to the typical scales of the problem. Some of the above operators such as $(\partial_\mu\varphi)^2 s^2$ and $(\partial_\mu s)^2\varphi^2$, along with $\frac{m^2_{\phi,s}}{M^2_{\rm Pl}}\phi^2 s^2$, are generated by the tree level graviton exchange [9]. Others can be generated at loop level and non–perturbatively. Since gravity is non–renormalizable, their coefficients should be treated as arbitrary input parameters. The above interactions are responsible for dark matter production after inflation, in particular, during the inflaton oscillation phase. Depending on the C_i coefficients, the production mechanism can be perturbative or non–perturbative (resonant).

To mention but one example, some of the above operators operators appear automatically in theories with non–minimal couplings of scalars to gravity [4]. In particular, $(\partial_\mu s)^2\varphi^2$, $(\partial_\mu\varphi)^2 s^2$ and $s^2 V(\varphi)$ are induced already at tree level by the metric transformation from the Jordan frame to the Einstein frame [14]. The (unsuppressed) operators $\varphi^2 s^4$ and $\varphi^4 s^2$ appear in these models at 1–loop via the graviton loop. Their coefficients are proportional to the product of the non–minimal couplings and the loop factor, and thus expected to be significant (in the absence of symmetry arguments). In general, it is a challenging task to estimate the relative size of the different operators since this can only be done reliably within UV complete gravity theories. On shell, two of the derivative operators can be eliminated via integration by parts:

$$(\partial_\mu\varphi)^2 s^2 \to (\partial_\mu s)^2\varphi^2 + m_\varphi^2\varphi^2 s^2 , \tag{4}$$

$$(\phi\partial_\mu\phi)(s\partial^\mu s) \to -\frac{1}{2}(\partial_\mu s)^2\phi^2 , \tag{5}$$

[1] The dim-5 operator $\varphi^3 s^2$ may in general be present, in which case it would dominate dark matter production.

where we have neglected the dark matter mass and the Hubble rate, which, during preheating is small compared to the particle energy. (We consider the Hubble–induced effects in Section 4). Focussing on dark matter pair production, we can thus restrict ourselves to the operators

$$O_3 = \frac{1}{M_{\rm Pl}^2} (\partial_\mu s)^2 \varphi^2 \, , \qquad O_4 = \frac{1}{M_{\rm Pl}^2} \varphi^4 s^2 \, , \tag{6}$$

amended with the renormalizable interaction

$$O_{\rm renorm} = \frac{m_\phi^2}{M_{\rm Pl}^2} \phi^2 s^2 \, . \tag{7}$$

Although this term is renormalizable, the coupling strength is highly suppressed: for typical inflaton masses it is below 10^{-10}. The operator $\varphi^2 s^4$ produces a final state with 4 DM quanta. The corresponding reaction rate is similar to that of the derivative operator, since the final state phase space gives analogous energy dependence. Therefore, we will not discuss this operator separately within the perturbative regime, while its non–perturbative analysis will be presented in Sec.4.

Clearly, operators O_3 and O_4 exhibit qualitatively different behaviour in regard to dark matter production. Indeed, O_3 involves the particle energy which is of the order of the (effective) inflaton mass, while in O_4 this dependence is replaced by the inflaton field value. The latter is not far from the Planck scale in typical models, thus

$$\phi \gg E_\phi \tag{8}$$

and one expects much more efficient DM production from O_4. In what follows, we make this argument more quantitative.

3 Perturbative dark matter production

An oscillating classical background can lead to particle production [11, 12, 13]. After inflation, φ oscillates coherently in either φ^2 or φ^4 potential, depending on the inflationary model. As a result, the $\varphi - s$ couplings induce dark matter pair production. If the corresponding coupling is small, the process can be described perturbatively. Below, we consider the contributions of the 3 basic operators to this reaction. We treat the Hubble expansion adiabatically such that the time dependence can be inserted in the inflaton oscillation amplitude at the end of the calculation. Also, we treat the produced dark matter particles as free and neglect backreaction. These approximations are justified at small inflaton–DM couplings.

3.1 $\varphi^2 s^2$ interaction

Consider the 4–point interaction

$$-\Delta L_{\text{renorm}} = \frac{1}{4}\lambda_{\phi s}\,\phi^2 s^2 \,, \tag{9}$$

where $\lambda_{\varphi s} \sim m^2{}_{\varphi}/M_{\text{Pl}}{}^2$. Let us expand the inflaton field as

$$\phi^2(t) = \sum_{n=-\infty}^{\infty} \zeta_n e^{-in\omega t} \,, \tag{10}$$

where the coefficients ζ_n are time–independent. Creation of a two–particle DM state with momenta p,q from the vacuum is described by the amplitude (in Peskin–Schroeder conventions [15])

$$-i\int_{-\infty}^{\infty} dt\langle f|V(t)|i\rangle = -i\,\frac{\lambda_{\phi s}}{2}\,(2\pi)^4\delta(\mathbf{p}+\mathbf{q})\sum_{n=1}^{\infty}\zeta_n\delta(E_p+E_q-n\omega) \,, \tag{11}$$

with $V(t)$ given by Eq.9. The corresponding invariant amplitude for the n-th inflaton mode decay is $\mathrm{M}_n = -\lambda_{\varphi s}\zeta_n/2$. The resulting reaction rate for DM pair production per unit volume is

$$\Gamma = \sum_{n=1}^{\infty}\Gamma_n = \sum_{n=1}^{\infty}\frac{1}{2}\int|\mathcal{M}_n|^2 d\Pi_n = \frac{\lambda_{\phi s}^2}{64\pi}\sum_{n=1}^{\infty}|\zeta_n|^2\sqrt{1-\left(\frac{2m_s}{n\omega}\right)^2}\,\theta(n\omega-2m_s) \,. \tag{12}$$

Here we have kept the DM mass for generality. The inflaton decay rate Γ_φ can be computed using energy conservation, $\rho_\varphi\Gamma_\varphi = \mathrm{h}E\mathrm{i}\Gamma$, where ρ_φ and $\mathrm{h}E\mathrm{i}$ are the inflaton energy density and the average energy of the decay products, respectively. Hence,

$$\Gamma_\phi = \frac{\lambda_{\phi s}^2\omega}{64\pi\rho_\phi}\sum_{n=1}^{\infty}n|\zeta_n|^2\sqrt{1-\left(\frac{2m_s}{n\omega}\right)^2}\,\theta(n\omega-2m_s) \,. \tag{13}$$

In the massless limit, one thus recovers the result of [16].

3.2 $\varphi^2(\partial_\mu s)^2$ interaction

The calculation proceeds as above, except the final state receives an additional momentum–dependent factor in the amplitude:

$$p\cdot q = \frac{1}{2}(p+q)^2 = \frac{1}{2}(E_p+E_q)^2 = \frac{1}{2}n^2\omega^2 \,,$$

where the DM mass has been neglected. So, effectively in the amplitude for the quartic interaction, one replaces $\lambda_{\phi s} \to 4C_3/M_{\text{Pl}}^2\, p\cdot q$, which in the $m_s \to 0$ limit leads to

$$\Gamma = \frac{C_3^2\,\omega^4}{16\pi M_{\text{Pl}}^4}\sum_{n=1}^{\infty}n^4|\zeta_n|^2 \,. \tag{14}$$

3.3 $\varphi^4 s^2$ interaction

The calculation is similar to that for the $\varphi^2 s^2$ case. φ^4 can be expanded as

$$\phi^4(t) = \sum_{n=-\infty}^{\infty}\hat{\zeta}_n e^{-in\omega t} \,, \tag{15}$$

where

$$\hat{\zeta}_n = \sum_{m=-\infty}^{\infty} \zeta_{n-m}\zeta_m \,. \tag{16}$$

Replacing $\lambda_{\varphi s} \to 4C_4/M_{\mathrm{Pl}}^2$, in the massless DM limit we get

$$\Gamma = \frac{C_4^2}{4\pi M_{\mathrm{Pl}}^4} \sum_{n=1}^{\infty} |\hat{\zeta}_n|^2 \,. \tag{17}$$

3.4 Relative efficiency

Let us estimate the relative particle production efficiency of the different operators. If the inflaton potential is quadratic, $V(\phi) = \frac{1}{2} m_\phi^2 \phi^2$, we have

$$\varphi(t) = \varphi_0 \cos m_\varphi t \,, \tag{18}$$

where φ_0 is the oscillation amplitude. (The fact that φ_0 decreases slowly in time, $\varphi_0 \propto 1/(m_\varphi t)$, is insignificant for our purposes). In the quartic case, $V(\phi) = \frac{1}{4}\lambda_\phi \phi^4$, the inflaton field is given by the Jacobi cosine,

$$\phi(t) = \phi_0 \,\mathrm{cn}\left(\sqrt{\lambda_\phi}\phi_0\, t, \frac{1}{\sqrt{2}}\right) = \frac{\sqrt{\pi}\,\Gamma\left(\frac{3}{4}\right)}{\Gamma\left(\frac{5}{4}\right)}\,\phi_0 \sum_{n=1}^{\infty}\left(e^{i(2n-1)\omega t} + e^{-i(2n-1)\omega t}\right)\frac{e^{-(\pi/2)(2n-1)}}{1+e^{-\pi(2n-1)}} \,, \tag{19}$$

where

$$\omega = \frac{1}{2}\sqrt{\frac{\pi}{6}}\frac{\Gamma\left(\frac{3}{4}\right)}{\Gamma\left(\frac{5}{4}\right)}\, m_\phi^{\mathrm{eff}} \tag{20}$$

and

$$m_\phi^{\mathrm{eff}} = \sqrt{3\lambda_\phi}\,\phi_0 \,. \tag{21}$$

For many purposes, the above sum can be approximated by the first term with $n = 1$.

The relative efficiency of $\mathcal{O}_3$ and $\mathcal{O}_4$ is given by

$$\frac{\Gamma[\mathcal{O}_3]}{\Gamma[\mathcal{O}_4]} \sim \frac{C_3^2}{C_4^2}\frac{\omega^4}{\phi_0^4} \,. \tag{22}$$

Clearly, the reaction rate due to $\varphi^2(\partial_\mu s)^2$ is much suppressed compared to that of $\varphi^4 s^2$. For the quadratic inflaton potential, the suppression factor is

$$m_\phi^4/\phi_0^4 \sim 10^{-20} \,, \tag{23}$$

assuming the typical values $\varphi_0 \sim 1$ and $m_\varphi \sim 10^{-5}$ in Planck units. In the quartic case, $\omega^4/\phi_0^4 \sim \lambda_\phi^2 < 10^{-20}$ for typical $\lambda_\varphi < 10^{-10}$.

The contribution of the $\varphi^2 s^2$–operator of the form (7) is similarly suppressed by m_ϕ^4/ϕ_0^4. It is also clear that, due to the phase space integral, the rate of the $\mathcal{O}_5$–induced process $\varphi\varphi \to ssss$ contains an additional factor of E_ϕ^4/ϕ_0^4 compared to the pair production rate from $\mathcal{O}_4$. Hence we conclude that the

$\varphi^4 s^2$ interaction dominates DM production, unless there is a large hierarchy in the Wilson coefficients, e.g. $C_3/C_4 \sim 10^{10}$.

The above calculation also tells us that higher dimensional operators

$$\frac{\tilde{C}_6}{M_{\rm Pl}^4}\phi^6 s^2 + \frac{\tilde{C}_8}{M_{\rm Pl}^6}\phi^8 s^2 + \ldots \tag{24}$$

are important. Indeed, their contributions are only suppressed by $\phi_0^4/M_{\rm Pl}^4$, etc. relative to that of O_4. If the inflaton amplitude is not far away from the Planck scale, this suppression is not very significant.

4 Resonant dark matter production via dim–6 operators

Perturbative calculations ignore the Bose enhancement of the amplitude due to the presence of identical states. Depending on the coupling, this enhancement can be very significant and lead to resonant production [12, 17, 18]. Such a regime can be described semiclassically by analyzing the equations of motion for the DM field s. In what follows, we compare the corresponding resonant particle production via operators O_3 and O_4.

To derive the EOM for s in the presence of the inflaton background $\varphi(t)$, consider the action

$$S = \int d^4x\sqrt{|g|}\left(\frac{1}{2}\,\mathcal{K}(\phi)\,g^{\mu\nu}\partial_\mu s\,\partial_\nu s - \mathcal{V}\right), \tag{25}$$

where the kinetic function $\mathcal{K}(\varphi)$ depends on φ only and $\mathcal{V}$ is the s*–dependent* part of the scalar potential. Here $g = \det g_{\mu\nu}$ and the Friedmann metric is $g_{\mu\nu} = \mathrm{diag}(1,-a^2,-a^2,-a^2)$. Since φ and a are functions of time only, variation of the action with respect to s yields

$$\ddot{s} - \frac{1}{a^2}\,\partial_i\partial_i s + \left(\frac{\dot{\mathcal{K}}}{\mathcal{K}} + 3H\right)\dot{s} + \frac{\mathcal{V}'_s}{\mathcal{K}} = 0\,. \tag{26}$$

Let us expand $s(t,\mathbf{x})$ in spacial Fourier modes $s_k(t)$, where k is the comoving 3–momentum. If V is quadratic in s, the different momentum modes decouple and we have

$$\ddot{s}_k + \left(\frac{\dot{\mathcal{K}}}{\mathcal{K}} + 3H\right)\dot{s}_k + \left(\frac{k^2}{a^2} + \frac{\mathcal{V}''_s}{\mathcal{K}}\right)s_k = 0\,. \tag{27}$$

Now suppose that the renormalizable potential vanishes and only one dim–6 operator is present at a time, such that either $\mathcal{K} = 1$ or $\mathcal{V} = 0$.

(a) Consider first the case of a non–trivial kinetic function $\mathcal{K}$ and $\mathcal{V} = 0$. The first order time derivatives can be eliminated by the change of variables

$$s_k = a^{-3/2}\mathcal{K}^{-1/2}X_k, \tag{28}$$

such that

$$\ddot{X}_k + \left[\frac{1}{4}\frac{\dot{\mathcal{K}}^2}{\mathcal{K}^2} - \frac{1}{2}\frac{\ddot{\mathcal{K}}}{\mathcal{K}} - \frac{3}{2}H\frac{\dot{\mathcal{K}}}{\mathcal{K}} + \frac{9}{4}wH^2 + \frac{k^2}{a^2}\right]X_k = 0\,, \tag{29}$$

where the equation of state coefficient is

$$w = -\left(1 + \frac{2\dot{H}}{3H^2}\right). \tag{30}$$

In the limit of a constant inflaton amplitude, the coefficients are periodic in time such that the above EOM belongs to the class of *Hill's equations*. Depending on the parameters, the solution X_k can grow exponentially in time signifying particle production.

Let us now specialize to operator O_3,

$$\mathcal{K} = 1 + \frac{2C_3}{M_{\rm Pl}^2}\,\phi^2. \tag{31}$$

The effective field theory expansion makes sense if $C_3\phi^2/M_{\rm Pl}^2 \ll 1$. Consider the *locally* quadratic inflaton potential such that $\varphi(t) = \varphi_0(t)\cos m_\varphi t$ with $\varphi_0 \propto 1/(m_\varphi t)$. In this case, the Universe is matter dominated, $w = 0$ and

$$H = \frac{m_\varphi \varphi_0}{\sqrt{6}M_{\rm Pl}}. \tag{32}$$

For $C_3 < 1$ and $\varphi_0 < M_{\rm Pl}$, the term $(\dot{\mathrm{K}}/\mathrm{K})^2$ in the square brackets is insignificant, as is wH^2. The terms $H\dot{\mathrm{K}}/\mathrm{K}$ and $\ddot{\mathrm{K}}/\mathrm{K}$ are similar in magnitude initially, but the former is cubic in φ_0 and thus decreases faster in time. Keeping just the $\ddot{\mathrm{K}}/\mathrm{K}$ term, we may approximate

$$\ddot{X}_k + \left[\frac{2C_3 m_\phi^2\phi_0^2}{M_{\rm Pl}^2}\cos 2m_\phi t + \frac{k^2}{a^2}\right] X_k = 0. \tag{33}$$

This has the form of the *Mathieu equation*,

$$X_k'' + [A + 2q\cos 4z]\, X_k = 0\,, \tag{34}$$

where $z = m_\varphi t/2$, the prime stands for differentiation with respect to z, and

$$q = \frac{4C_3\phi_0^2}{M_{\rm Pl}^2}\quad,\quad A = \frac{4k^2}{a^2 m_\phi^2}. \tag{35}$$

A large q generally implies fast amplitude growth and efficient particle production via broad parametric resonance [17]. In our case, however, $q \ll 1$ and the resonance is narrow. As a result, no efficient particle production is possible, especially in view of the redshifting of the produced particle momenta [17],[19].

(b) Let us now consider the effect of O_4, so we take $\mathrm{K} = 1$ and

$$\mathcal{V} = \frac{C_4}{M_{\rm Pl}^2}\,\phi^4 s^2. \tag{36}$$

For the quadratic inflaton potential, the corresponding X_k satisfies

$$\ddot{X}_k + \left[\frac{2C_4\phi_0^4}{M_{\rm Pl}^2}\cos^4 m_\phi t + \frac{k^2}{a^2}\right]X_k = 0\,. \tag{37}$$

Using $\cos^4 x = \frac{1}{8}\cos 4x + \frac{1}{2}\cos 2x + \frac{3}{8}$, one can bring the EOM into the form of the *Whittaker–Hill equation*,

$$X_k'' + [A + 2p\cos 2z + 2q\cos 4z]X_k = 0\,, \tag{38}$$

where now $z = m_\varphi t$ and

$$q = \frac{C_4\phi_0^4}{8m_\phi^2 M_{\rm Pl}^2}\,,\quad p = \frac{C_4\phi_0^4}{2m_\phi^2 M_{\rm Pl}^2}\,,\quad A = \frac{k^2}{a^2 m_\varphi^2} + \frac{3C_4\phi_0^4}{4m_\varphi^2 M_{\rm Pl}^2}\,. \tag{39}$$

The efficiency of particle production is characterized by p and q, whose large values (depending on A) generally lead to broad resonance [20],[21]. We see that this regime is easily achieved in the presence of O_4. In particular, $p, q \gg 1$ is consistent with $C_4 \ll 1$ and sub–Planckian φ_0 values, as long as $\phi_0 \gg m_\phi$.

We therefore conclude that O_4 is much more efficient in particle production than O_3. The same conclusion applies to the quartic inflaton potential: the analysis proceeds analogously up to the replacement of the inflaton mass with the effective inflaton mass $m^{\rm eff}_\varphi \sim \sqrt{\lambda_\varphi}\varphi_0$.

The resonance efficiency is determined by the ratio of the inflaton–induced DM mass to the inflaton effective mass. In the list (3), this ratio can be large only for operator O_4. Indeed, $(\partial_\mu\varphi)^2 s^2$ induces the DM mass of order m_φ at best.[1] The term $(\varphi\partial_\mu\varphi)(s\partial^\mu s) = (\partial_\mu\varphi^2)(\partial^\mu s^2)/4$ can be rewritten as a mass term for s by integrating by parts. The resulting DM mass scale is determined by m_φ or H, leading to the same conclusion. The operator $\varphi^2 s^4$ does not induce any mass in our approximation and the corresponding EOM is not of Hill's type (see below).[2] Finally, the renormalizable term $\varphi^2 s^2$ of the form (7) does not lead to broad resonance since the corresponding $q \lesssim 1$. It is thus clear that O_4 dominates particle production.

Similar conclusions apply to higher dimensional operators $\varphi^6 s^2$, etc. As long as φ_0 is not much below the Planck scale, the effective q parameter can be much greater than one, signifying efficient particle production. Therefore, the results are sensitive to the presence of operators of this type.

The amount of dark matter produced during preheating is difficult to estimate analytically. The reason is that the parameters of the Whittaker–Hill equation evolve in time making the resonance stochastic, which is further complicated by the non–trivial 3-D stability band structure [21]. Depending on the size of C_4, tangible backreaction and rescattering effects [22],[23] can also take place. Thus, to make reliable predictions, we have to resort to lattice simulations.

4.1 On resonant production via $\varphi^2 s^4$

Unlike for other operators considered in this work, resonant particle production via $O_5 = \varphi^2 s^4$ is not described by the Hill's equation. In this case, the system is non–linear already to leading order and thus

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[1] The induced mass term appears when the variance $\langle s^2\rangle$ becomes significant.

difficult to handle analytically. In this subsection, we discuss some of its properties relevant to the subject of our paper.

The EOM for the DM field s in the presence of O_5 reads

$$\ddot{s} - \frac{1}{a^2}\,\partial_i\partial_i s + 3H\,\dot{s} + \frac{4C_5\phi^2(t)}{M_{\rm Pl}^2}\,s^3 = 0\,. \tag{40}$$

Clearly, the different momentum modes do not decouple in this case. As a representative example, let us focus on the zero mode of s which normally plays a major role in particle production. Omitting the gradient term and introducing a rescaled field X ($s = a^{-3/2}X$), we get

$$\ddot{X} + \frac{9}{4}wH^2\,X + \frac{4C_5\phi^2(t)}{M_{\rm Pl}^2\;a^3}\,X^3 = 0\,, \tag{41}$$

where w is the coefficient of the equation of state of the system. Let us now specialize to the quadratic inflaton potential such that $w = 0$,

$$\frac{a}{a_0} = \left(\frac{m_\phi t}{m_\phi t_0}\right)^{2/3}\,, \tag{42}$$

and $\phi(t) \simeq 1.85\,\varphi_0 \frac{\cos m_\phi t}{m_\phi t}$. Here the initial condition is chosen such that $\varphi(t_0) = \phi_0$ with $m_\varphi t_0 = 1$. The above EOM should be supplemented by boundary conditions. The magnitude of a light field is given by quantum fluctuations, such that we may take $s \sim H$ and $\dot{s} \sim H^2$ initially, where $H = m_\varphi\phi_0/(\sqrt{6}M_{\rm Pl})$. Introducing

$$z = m_\varphi t\,,\; Y = X/m_\varphi\,, \tag{43}$$

we get

$$Y'' + \kappa\,\frac{\cos^2 z}{z^4}\,Y^3 = 0\,, \tag{44}$$

where the prime denotes differentiation with respect to z. The representative boundary conditions can be chosen as $Y(1) = 1, Y'(1) = 1$. The coupling κ is given by

$$\kappa \simeq \frac{14C_5\varphi_0^2}{M_{\rm Pl}^2\,a_0^3}\,, \tag{45}$$

where typically $\phi_0 \sim M_{\rm Pl}$, $a_0 \sim 1$ and the effective field theory approach is expected to be valid for $\kappa \lesssim O(1)$.

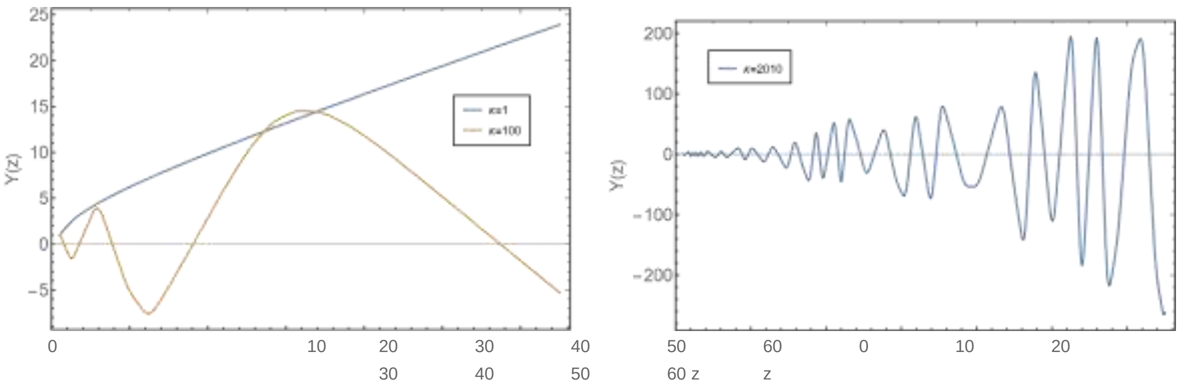

Figure 1: Solutions to Eq.44 for $\kappa = 1{,}100{,}2010$.

The oscillating term in Eq.44 falls faster with time then the analogous coefficient in the Mathieu equation, hence the duration of the resonance is shorter in the present case. The equation exhibits a simple asymptotic solution for $z \gg 1$ (or $\kappa \ll 1$),

$$Y \propto z\,. \tag{46}$$

On the other hand, for $\kappa \gg 1$, $Y(z)$ varies much faster than $\cos^2 z/z^4$ does such that the latter can be treated adiabatically. In this case, Eq.44 takes the form $Y'' + cY^2 = 0$ with $c = \kappa \cos^2 z/z^4$, whose solution is a Jacobi cosine. Given the amplitude of oscillations Y_0, locally we have

$$Y(z) \simeq Y_0\, \mathrm{cn}\left(\sqrt{c}\, Y_0 z, \frac{1}{\sqrt{2}}\right) \tag{47}$$

in the convention of [18]. The oscillation frequency $\sqrt{c}Y_0$ changes non–adiabatically around the inflaton zero crossings, implying particle production.

Numerical solutions to Eq.44 are presented in Fig.1. For $\kappa = 1$, the solution quickly takes on the asymptotic form (46), which corresponds to constant s in our approximation ($m_s \sim 0$). Even for $\kappa = 100$, there is no significant amplitude growth, while for $\kappa = 2010$ the solution exhibits truly resonant behaviour.[3] We thus find that κ & 10^3 is necessary for efficient DM production.

The discussion above concerns the zero mode. Normally it serves as an indicator whether or not resonant particle production takes place. To reaffirm it, we have performed lattice simulations of the full inflaton–DM system in the regime $C_5\phi^2/M_{\rm Pl}^2 \lesssim 1$ and indeed found no tangible particle production.

We therefore conclude that no efficient particle production is induced by operator O_5 as long as $C_5\phi^2/M_{\rm Pl}^2 \lesssim \mathcal{O}(1)$.

5 Lattice simulations: reproducing the correct DM abundance

In this Section, we focus on the leading operator $\frac{C_4}{M_{\rm Pl}^2}\phi^4 s^2$ and compute the resulting DM relic

[2] We find that the solution in this regime is quite sensitive to the numerical method used for solving the differential equation.

abundance in the resonant regime, $C_4\phi_0^4/M_{\rm Pl}^2 \gg m_\phi^2$, assuming $m_\phi \gg m_s$. As explained above, the analytical approach to resonant particle production has significant limitations, given the complexity of the Whittaker–Hill equation as well as backreaction and rescattering effects. Therefore, we resort to lattice simulations using the numerical tool CosmoLattice [24, 25].

A realistic framework must also account for the Standard Model particle production. The simplest way to incorporate reheating is to include a small Higgs–inflaton coupling following [26, 27],

$$V_{\varphi h} = \sigma_{\varphi h}\varphi H^\dagger H\,, \tag{48}$$

which would lead to late–time decay of the inflaton into the Higgs pairs (for $m_\varphi > 2m_h$). As long as $\sigma_{\varphi h}$ is sufficiently small in Planck units, resonant dark matter production is not affected by this coupling. The dark matter abundance is expressed in terms of

$$Y = \frac{n}{s_{\rm SM}}\,, \quad s_{\rm SM} = \frac{2\pi^2}{45} g_{*s} T^3\,, \tag{49}$$

where n is the DM number density, $s_{\rm SM}$ is the Standard Model entropy density at temperature T and g_{*s} is the effective number of SM degrees of freedom contributing to the entropy. We are interested in very weakly interacting dark matter such that it never reaches thermal equilibrium with the environment. After preheating ends, the total number of the DM quanta remains constant. Since the SM entropy is also conserved, Y can be computed at the reheating stage. The observed value is [28]

$$Y_\infty = 4.4 \times 10^{-10} \left(\frac{\rm GeV}{m_s}\right)\,, \tag{50}$$

which sets a constraint on the model parameters.

Reheating occurs when

$$H_R \simeq \Gamma_{\phi\to hh}\,, \quad \Gamma_{\phi\to hh} = \frac{\sigma_{\phi h}^2}{8\pi m_\phi}\,, \tag{51}$$

where H_R is the Hubble rate at reheating and $\Gamma_{\varphi\to hh}$ takes into account 4 Higgs d.o.f. at high energies. The reheating temperature is given by

$$H_R = \sqrt{\frac{\pi^2 g_*}{90}}\,\frac{T_R^2}{M_{\rm Pl}}\,, \tag{52}$$

where g_* is the effective number of the Standard Model degrees of freedom contributing to the energy density. Combining T_R with the dark matter density n computed on the lattice, one determines Y according to (49).

The resulting abundance is sensitive to the energy balance between the inflaton and dark matter, which affects the expansion history. Depending on the coupling and the inflaton potential, DM can contribute a significant fraction up to 50% to the total energy density at the end of preheating. We therefore parametrize

$$\rho_e(s) = \delta\, \rho_e(\varphi)\,, \tag{53}$$

where $\rho_e(s)$, $\rho_e(\varphi)$ are the DM and inflaton energy densities, respectively, evaluated at the end of the simulation. At weak coupling, in the quadratic inflaton potential we have $\delta \sim 0$, while, at strong coupling, δ can reach a value close to 1. The Universe evolution proceeds in stages: first, it can be dominated by radiation; later, when the energy per quantum becomes comparable to the inflaton mass, it evolves as non–relativistic matter; finally, the Universe reheats and becomes radiation–like. Denoting the corresponding scale factors as a_e (end of the simulation), a_* (transition), a_R (reheating), we have

$$a_e \xrightarrow{\mathrm{rel}} a_* \xrightarrow{\mathrm{nrel}} a_R, \tag{54}$$

such that the Hubble rate evolves as $H \sim a^{-3(w+1)/2}$ with $w = 1/3$ and $w = 0$ during the two periods, respectively. After the transition point a_*, $\rho(s)$ becomes negligible and at a_R the inflaton energy density $\rho(\varphi)$ converts into SM radiation. Thus,

$$H_R = \frac{H_e}{\sqrt{1+\delta}} \frac{a_e^2}{a_*^2} \frac{a_*^{3/2}}{a_R^{3/2}}, \tag{55}$$

where H_e is the Hubble rate at the end of the simulation. We note that the first stage of the radiation–like expansion may collapse to a point, i.e. $a_e = a_*$. This is the case for the quadratic inflaton potential at weak inflaton–DM coupling.

Solving for a_R, we find $\sigma_{\varphi h}$ required by the correct DM abundance in terms of the simulation output:

$$\sigma_{\varphi h} \simeq 1.6 \times 10^{-8} \sqrt{m_\varphi M_{\mathrm{Pl}}^3}\, \frac{H_e^2}{(1+\delta)\, n_e} \frac{a_e}{a_*} \frac{\mathrm{GeV}}{m_s} \tag{56}$$

for $g_* \simeq 107$ and M_{Pl} being the reduced Planck mass. The values of H_e, n_e, δ at the end of preheating are computed by CosmoLattice, while a_e/a_* can be determined by tracking the equation of state of the system.

This formula can be simplified further if we define a_* according to

$$\langle E_e(\phi)\rangle \frac{a_e}{a_*} \simeq m_\phi, \tag{57}$$

where the average energy of the inflaton quantum at the end of the simulation is $\langle E_e(\varphi)\rangle = \rho_e(\varphi)/n_e(\varphi)$. This definition of a_* is more practical in that it does not require tracking the equation of state of the system over a long period, which is computationally challenging. We then get

$$\sigma_{\varphi h} \simeq 5 \times 10^{-9} \frac{m_\phi^{3/2}}{M_{\mathrm{Pl}}^{1/2}} \frac{n_e(\phi)}{n_e(s)} \left(\frac{\mathrm{GeV}}{m_s}\right), \tag{58}$$

which only requires the particle densities as an output of the simulations. Here $n_e(\varphi)$ includes the inflaton quanta with zero momentum and typically $n_e(\phi) \gg n_e(s)$ unless the coupling is relatively strong.

The number densities are computed via the k–mode occupation numbers n_k. For the dark matter field, we have

$$n_k = \frac{\omega_k}{2}\left(\frac{|\dot{X}_k|^2}{\omega_k^2} + |X_k|^2\right) - \frac{1}{2}, \tag{59}$$

where $\omega_k^2(t) = \frac{2C_4\phi_0^4}{M_{\rm Pl}^2}\cos^4 m_\phi t + \frac{k^2}{a^2}$ in the quadratic inflaton potential. Here X_k is a solution to the EOM with the boundary condition given by quantum fluctuations. The resulting number density is then given by

$$n(s) = \frac{1}{(2\pi a)^3}\int d^3k\, n_k. \tag{60}$$

On the lattice, the momentum spectrum is discrete which allows one to treat the zero mode separately. Analogous formulae apply to the inflaton field and the quartic potential. It is important to remember that the EOM for the different momentum modes decouple at weak couplings only. The lattice approach allows us to incorporate the couplings among the k–modes of the inflaton and DM, thereby accounting for backreaction and rescattering. The latter can have a crucial impact on the dynamics of the system (see [29, 30] for recent examples).

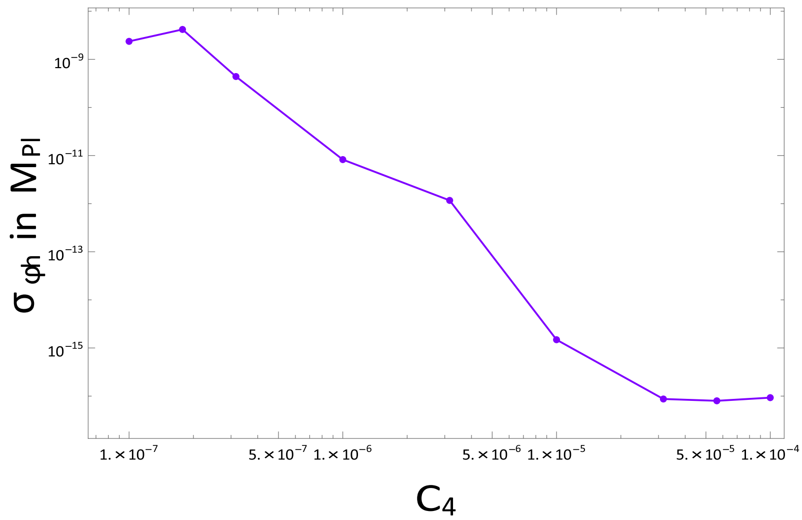

Figure 2: C_4 vs $\sigma_{\varphi h}$ required for the correct DM abundance in the quadratic inflaton potential ($m_\varphi = 10^{13}$ GeV, $m_s = 1$ GeV, ϕ_0 ' $M_{\rm Pl}$). The simulations are performed with CosmoLattice [24].

Our numerical results are presented in Fig.2. The correct relic density is produced in a wide range of C_4 between 10^{-7} and 10^{-4}. The lower bound comes from requiring semiclassical behaviour, that is, the occupation numbers must be sufficiently large. The upper bound has technical nature: the simulation becomes unstable. We observe that the curve tends to flatten out at larger couplings. This is expected from quasi–equilibrium: as $n_e(\varphi)$ approaches $n_e(s)$, the $\sigma_{\varphi h}$ coupling becomes constant within our approximation [27]. Although such flattening is clearly visible, we find that quasi–equilibrium has not yet been reached at $C_4 \sim 10^{-4}$. We estimate the required C_4 to be of order 10^{-3}, yet the simulations in this range become less reliable.

Given these results, we can now determine under what circumstances the renormalizable coupling $\frac{1}{4}\lambda_{\phi s}\phi^2 s^2$ becomes unimportant. According to Fig.4 of [27], $\lambda_{\varphi s} < 10^{-8}$ does not make any significant contribution to the dark matter abundance in the parameter range of interest. This can be understood intuitively since $\lambda_{\varphi s}/4 < C_4\phi_0^2/M_{\rm Pl}^2$ in this case (see also [20]). On the other hand, for $\lambda_{\varphi s} > 10^{-7} - 10^{-6}$ or $C_4 > 10^{-3}$, the inflaton–DM system reaches quasi– equilibrium and the DM abundance becomes independent of these couplings, with the required $\sigma_{\varphi h} \sim 10^{-17} M_{\rm Pl}$.

Subsequently, when the inflaton coherence is lost, the operator $\mathcal{O}_4$ relinquishes its privileged role in DM production. In quasi–equilibrium, the scattering processes $\varphi\varphi \to ss$ can become comparably significant, depending on the corresponding C_i. Since all of our operators are Planck– suppressed, such processes are slower than the Hubble rate and thus do not lead to inflaton thermalization [31] nor significant DM production. At weak couplings, the inflaton field may remain semi–classical during reheating, in which case the effects of the SM thermal bath can be included along the lines of [32],[33].

Throughout this work we assume that other sources of dark matter are subdominant. In particular, we neglect the DM coupling to the Higgs such that no tangible freeze–in contribution from the Higgs thermal bath appears. This approximation is justified if this coupling is below 10^{-11} [34]. Furthermore, as explained in the Introduction, there is also a truly gravitational source of DM: the Universe expansion. However, in the presence of $\mathcal{O}_4$, the effective mass of s during inflation can be larger than the Hubble rate due to super–Planckian inflaton values, which suppresses $\langle s^2 \rangle$ and makes this production mechanism inefficient.

In our analysis, we have relied on the effective field theory expansion in the Einstein frame, which is expected to be meaningful during preheating. Gravitational dark matter production can also be encoded in the DM non–minimal coupling to gravity [35], which corresponds to a specific choice of higher dimensional operators in our approach. A related option is provided by gravity–induced inflaton decay in Starobinsky–like models [36].

6 Conclusion

In this work, we have studied perturbative and non–perturbative dark matter production during the inflaton oscillation epoch. We focus on the regime where the renormalizable interactions between the inflaton and dark matter are negligible. To determine the leading contributions, we resort to the effective field theory expansion in the inverse Planck mass. Such higher dimensional operators are expected to be generated by perturbative or non–perturbative gravitational effects. In the absence of quantum gravity theory, their coefficients are unknown and therefore treated as arbitrary input parameters.

We have focussed on Planck–suppressed dim–6 operators and studied their relative importance in the perturbative and resonant regimes. We find that operators of the form $\varphi^n s^2$ ($n \geq 4$) by far dominate particle production. They can generate the correct (non–thermal) dark matter abundance even for small values of the corresponding Wilson coefficients. Therefore, the phenomenological frameworks describing dark matter production are sensitive to the presence of such operators, which reinforces the importance of gravitational effects in this context.

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